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Density Variation and Isopycnic Layers

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[illegible] pycnic Layers

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ABSTRACT

A pole-to-pole study of density deviations near the 80th meridian west is presented from the surface to 31 km altitude. Density deviations are greatest at the surface, and under extreme conditions may range from 1.0 to 2.0 kg m^{-3}. Density decreases almost exponentially with altitude and occasionally falls below 0.01 kg m^{-3} at 31 km. Density deviations decrease from the surface to an isopycnic layer, which varies in height from 6 km in polar regions to 12 km at the equator. Above this isopycnic layer, density variations increase with altitude to a maximum density deviation layer. This maximum density deviation layer occurs along the base of the summer tropopause and is approximately the center of the tropospheric wind maximum. The maximum density deviation layer is parallel to, and 50 per cent higher in altitude than the lower isopycnic layer. A weaker, second isopycnic layer is shown above and parallel to the maximum density deviation layer; this second isopycnic layer is found in tropical regions and near the south pole. Because of large seasonal and latitudinal variations in atmospheric density, no single standard atmosphere can present density data adequate for high speed vehicle operations on a global basis.

1. Introduction

Air density has always been of somewhat secondary interest to meteorologists. But in recent years aviation and missile designers have become aware of its critical importance, for air density is one of the most important factors governing the passage of high velocity bodies through the atmosphere. Thrust, dynamic pressure, aerodynamic drag, vibration, structural and guidance limitations, and heating during the re-entry phase are factors limiting vehicle performance which are directly related to air density.

A number of density studies are available, but the meteorological literature is less voluminous for density than for temperature or wind, although several studies have been made in recent years for the military and space agencies. The improved quantity, quality, and altitude of atmospheric observations in recent years made a new study of air density with its range of variability desirable. The investigation of density variation leads immediately to the study of isopycnic layers (layers of minimum density variation) and layers of maximum density variation.

2. Data

To obtain the most representative data available, International Geophysical Year (IGY) radiosonde observations made near 80W were obtained from the National Weather Records Center at Asheville, N. C. Only the 1200 GMT observations were used. The observations were made during a three year period from 1957 to 1960, but for several stations, particularly in the Southern Hemisphere, less than three years of data are available. Also, observations decrease in number with altitude. Hence data above 18 km may be biased, particularly in polar regions.

In view of the small number of observations available in the International Geophysical Year program, radiosonde data were obtained from Cape Kennedy (Atlantic Missile Range), Florida, for a more detailed study of a single station. The observations were made during the period 1951–1957, inclusive, and here two observations per day were used. Observations were made at Patrick Air Force Base through 17 November 1956, and at Cape Kennedy thereafter. Since the stations are only 24 km apart, no significant differences in density could be ex-

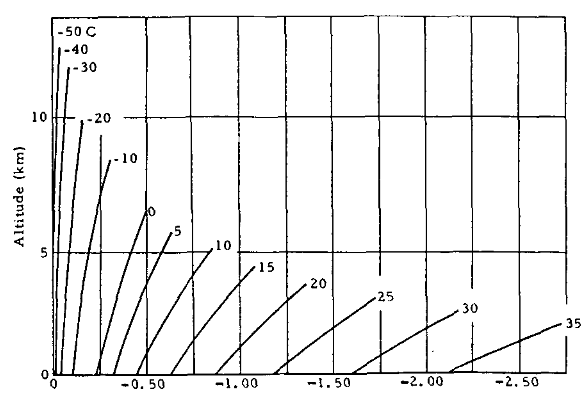

FIG. 1. Variation of density with respect to altitude and temperature at 100 per cent relative humidity.

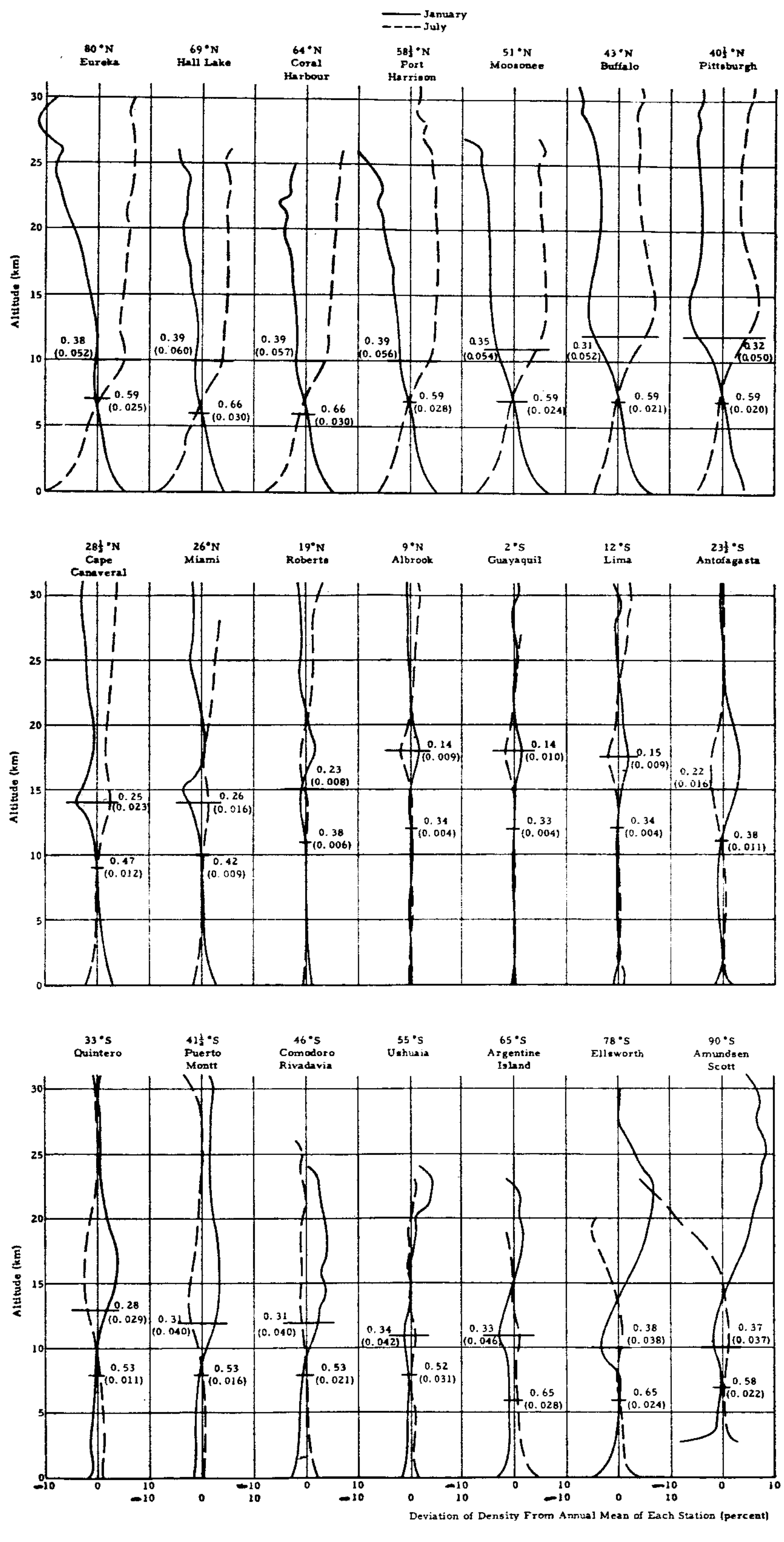

FIG. 2. Relative deviation of density from the annual mean at each station. (Mean annual density and ±3-standard deviations entered at the first isopycnic level and at the level of maximum deviation).

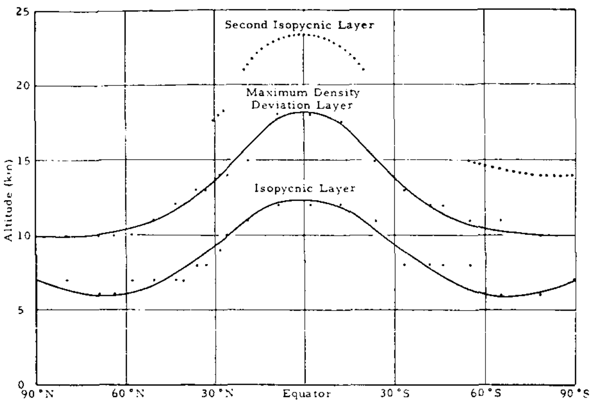

FIG. 3. The isopycnic layers and layer of maximum density deviations.

pected between these two stations, and in this study the combined data will be attributed to Cape Kennedy. At Cape Kennedy almost 5000 observations were available up to 10 km altitude; above 10 km the number of observations decreases rapidly with altitude to only 777 observations at 30 km.

Since air density is not usually measured directly, it is determined from pressure, temperature and humidity measurements. (See *Smithsonian Meteorological Tables* edited by List, 1958.) Density was computed by the formula:

$$\text{Density} = [0.3486(P - 0.377 e_s RH)]/T, \text{ where}$$

T is temperature, K
P is pressure, mb
RH is relative humidity, per cent
e_s is saturation vapor pressure, mb.

All of the data were put on punch cards and checked electronically to eliminate obvious errors. No corrections to temperature were made for radiation errors.

Density decreases as humidity increases, as shown in Fig. 1 (constructed from Smithsonian data plus author's extrapolations). But this decrease will rarely be as much as $2\frac{1}{2}$ per cent, and outside the tropics, humidity seldom causes density to decrease by as much as 1 per cent and that only near the surface. Due to the decrease of moisture with altitude, humidity may be neglected in density computations above 10 km. Pressure and temperature then are the primary factors which govern density.

Because of the large vertical variation, absolute density is difficult to depict graphically over this 31-km range so relative deviations of density about the mean annual average at each station have been plotted in Figs. 2 and 4–8; annual medians were used in Figs. 9–11. To enable the reader to convert readily to absolute density values, the mean annual density has been entered at 2-km intervals on the right of Figs. 4–8. The density entry near zero altitude is the mean annual surface value and not sea level density.

The mean monthly and annual averages and the standard deviations of density were computed for each kilometer of altitude above sea level. The median density values at Cape Kennedy (Fig. 9) show the isopycnic layer to occur at $10\frac{1}{2}$ km instead of at 9 km as shown by the Cape Kennedy monthly mean densities in Fig. 2. This is due to a non-Guassian distribution of density. Otherwise the Cape Kennedy median density values agree well with the mean density values.

3. Selecting the isopycnic level

The *Glossary of Meteorology* (Huschke, 1959) defines the isopycnic level as, "Specifically, a level surface in the atmosphere, at about 8 km altitude, where the air density is approximately constant in space and time." At all stations in this study, absolute density variations are greatest at the surface and decrease with altitude up to an isopycnic level. A study of density by Sen (1924) was the first to show an isopycnic level near 8 km with opposing changes in the seasonal density distribution above and below, although he called it the thermopause. Humphreys (1940) attempted to show mathematically that a level of constant density must exist near 8 km altitude at all seasons in all parts of the world. This conclusion was generally accepted, and it has remained unchallenged for more than 30 years. Sissenwine, Ripley and Cole (1958), Cole (1961), Cole and Court (1962), Whitehead, Pitts and Blick (1963), and others who have studied density have generally accepted this conclusion although Whitehead, Pitts and Blick (1963) show mathematically that other isopycnic heights may occur. The constant isopycnic level was accepted because most of the basic data for these density studies came from latitudes where the mean height of the isopycnic layer does average near 8 km. However, the relative deviations of density near 80W, shown in Fig. 2, and height plots of the maximum and minimum deviations of absolute density, shown in Fig. 3, present a different picture. Fig. 3 shows that the height of the isopycnic layer rises from near 6 km at 70 deg latitude to 12 km at the equator, and from 70 deg to the poles it apparently rises to 7 km altitude, thus describing a bow-shaped curve which needs very little smoothing even from the rather scant International Geophysical Year data. This pattern in the layer of minimum density deviations may also be traced in Fig. 78 of the 80W cross sections of Smith, McMurray and Crutcher (1963), whose cross sections are recommended for further details of global temperature, pressure and density distributions.

In the equatorial region, where temperature changes are small, density changes are also small. Based on Fig. 2, a thick isopycnic zone might reasonably be said to exist there from about 5–14 km. From Ushuaia south, it is difficult to choose an isopycnic level from the plots of relative density deviation. Partially to avoid these

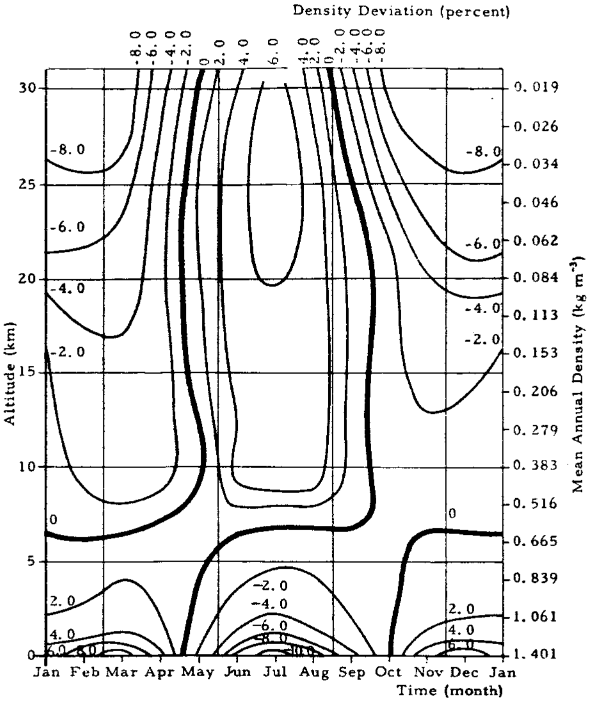

FIG. 4. Relative deviation of density from the mean annual density at Eureka, Northwest Territories.

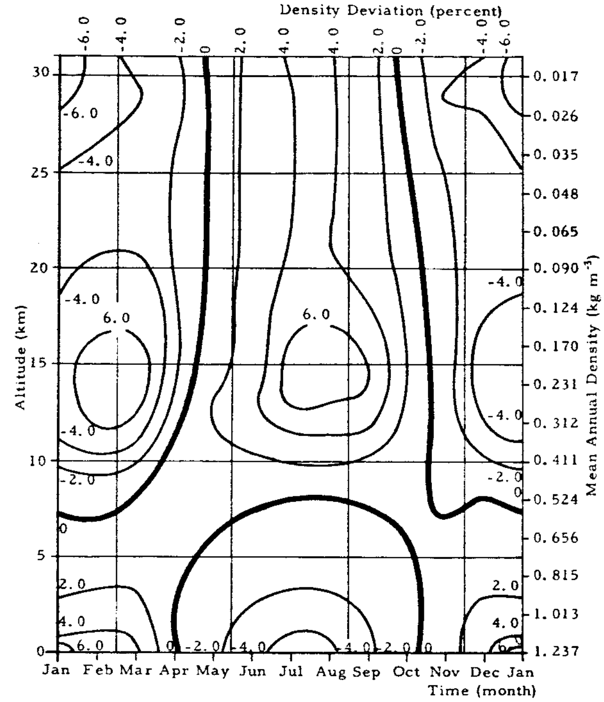

FIG. 5. Relative deviation of density from the mean annual density at Buffalo, N. Y.

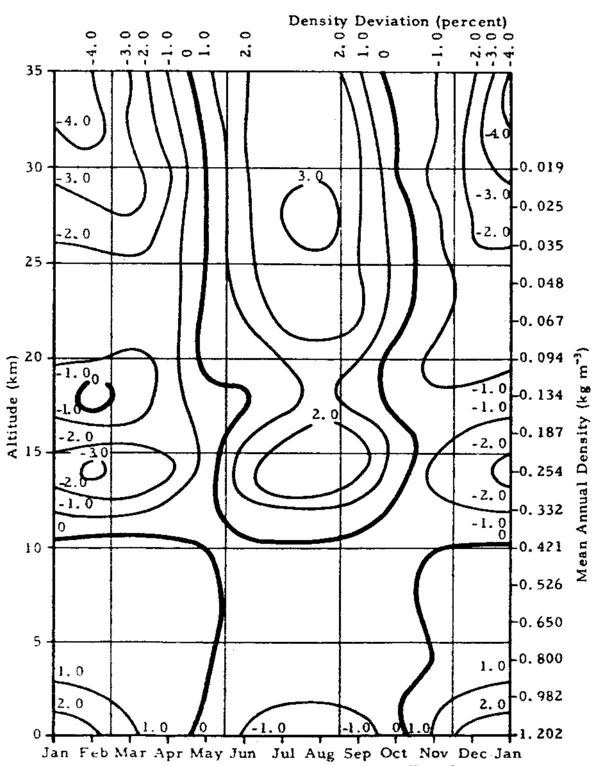

FIG. 6. Relative deviation of density from the mean annual density at Cape Kennedy (Canaveral), Fla.

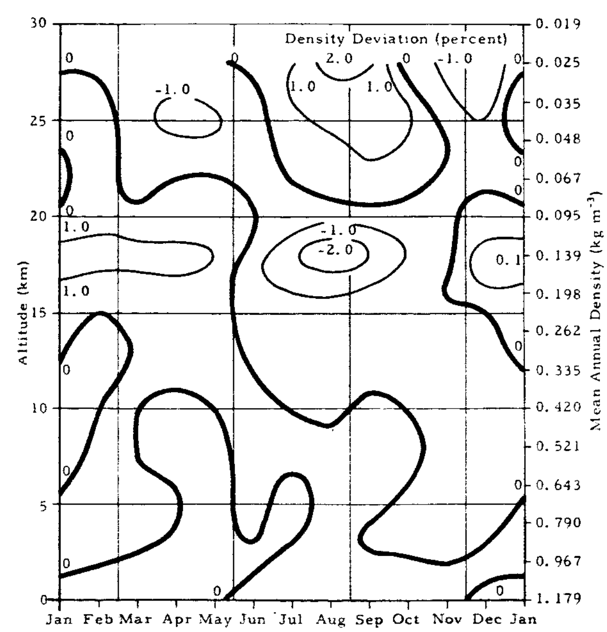

FIG. 7. Relative deviation from the mean annual density at Guayaquil, Equador.

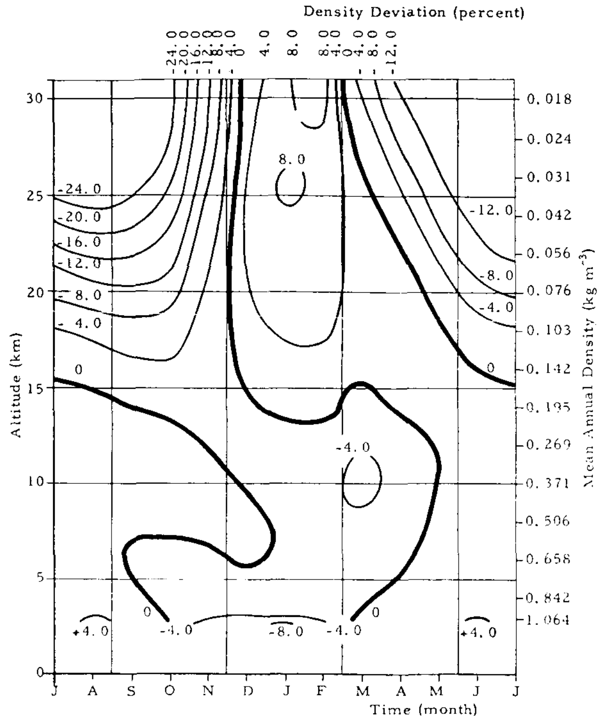

FIG. 8. Relative deviation of density from the mean annual density at Amundsen-Scott, Antarctica.

difficulties, the isopycnic level was placed at that altitude where the standard deviation of absolute density is least (Fig. 3). The layers of maximum and minimum density deviations, shown in Fig. 3, cannot be exactly traced in Fig. 2. Fig. 3 is based on standard deviations of density which have been computed for each station from the annual data, whereas Fig. 2 shows relative deviations for January and July only. An isopycnic level selected in this way should be more representative than an isopycnic level based only on January and July data. The minimum value of the standard deviations usually, though not always, occurs near the intersection of the winter and summer plots of relative density deviation. The isopycnic layer of the Southern Hemisphere is virtually a mirror image of that of the Northern Hemisphere. Since the isopycnic surface varies greatly in altitude, it might better be described as a "layer" than as a "level." Data from any single International Geophysical Year station might be questioned because of the short period of record, but the uniform pattern made from the isopycnic layers of 23 stations establishes it beyond reasonable doubt.

The isopycnic layer is not a layer of zero density deviation, as the definition of the term might lead one to expect. In most cases it is merely the level where plots of winter and summer density approach each other or intersect, and the daily observations of density show a rather wide scatter around the mean annual average. In Fig. 2, the mean annual density and 3-standard deviations of density (in parentheses) are entered at both the isopycnic layer and at the layer of maximum density deviations.

While no single altitude can be said to be truly isopycnic at all latitudes in all seasons, near approaches to an isopycnic layer occur in spring and in fall when the polar temperatures are most nearly equalized. In April and in September, density in the 6-8 km region is virtually constant from pole-to-pole. In the 25–28 km region, density is nearly constant from December through February at all latitudes. At other times and altitudes density varies markedly along any fixed altitude level (see Figs. 2, 3 and 12). Contrary to the *Glossary of Meteorology* and the statements of Humphreys (1940) and others, density does vary along the isopycnic layer, both with time and latitude. The mean annual density of the isopycnic layer in the equatorial region, 0.33 kg m^{-3}, is only about half that near 70 deg latitude, i.e., 0.66 kg m^{-3} at Hall Lake and 0.65 kg m^{-3} at Argentine Island.

Except for the unfortunate finding of a constant height isopycnic level, Humphreys' (1940) global description of seasonal density changes is a model of conciseness and clarity so far as his data extended. And it must be conceded that density changes are relatively small at 8 km although they are still smaller at 6 km and at 7 km (see Fig. 12).

4. The layer of maximum density deviation

A layer of maximum density deviation occurs above and parallel to the isopycnic layer and at a 50 per cent higher altitude; but the available data do not show an altitude rise at the poles similar to the rise made by the isopycnic layer. Thus the layer of maximum density deviation varies from 18 km at the equator to 10 km in polar regions. This differs somewhat from the findings of Cole and Court (1962) who reported the greatest seasonable variability at 15 km altitude. Whitehead, Pitts and Blick (1963) also state that a level of maximum density variability occurs at 15–16 km. For a comparison of actual seasonal and latitudinal density changes at 8 and at 15 km see Fig. 12. From 60 deg latitude northward, seasonal density changes are as large at 8 km as at 15 km although the density range elsewhere is much greater at 15 km.

Since temperature rises from the poles to the equator, both the isopycnic and maximum density deviation layers can be said to rise as the surface temperature increases, and both layers closely parallel the base of the minimum temperature wedge described by Smith (1963). The base of this minimum temperature wedge is coincident with the summer tropopause. It is probably more than a coincidence that the layer of maximum density deviations is virtually identical with the summer tropopause. Another interesting observation is that the mean annual density of the layer of maximum den-

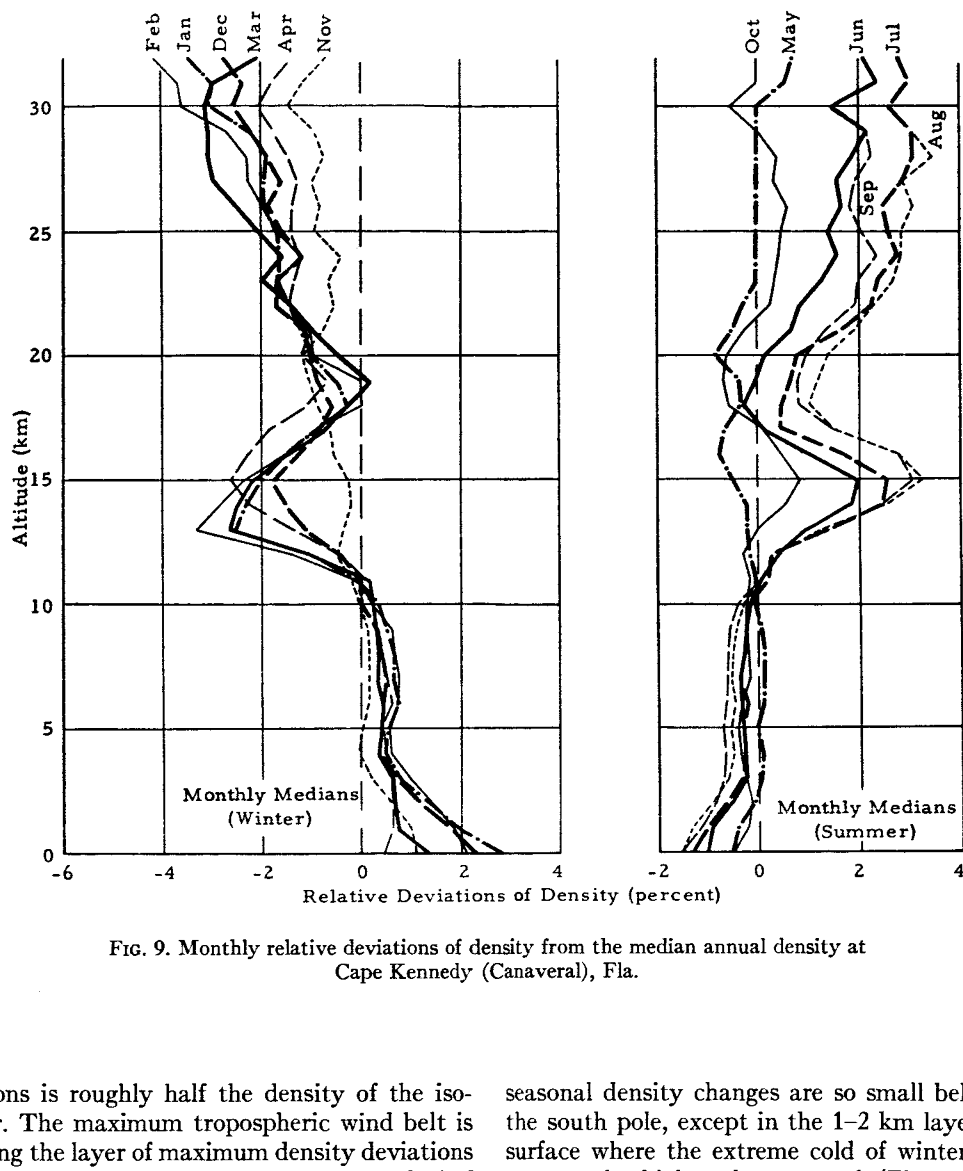

FIG. 9. Monthly relative deviations of density from the median annual density at Cape Kennedy (Canaveral), Fla.

sity deviations is roughly half the density of the isopycnic layer. The maximum tropospheric wind belt is centered along the layer of maximum density deviations as can be seen from the *Monthly Mean Aerological Cross Sections*, published by the U. S. Weather Bureau (1961).

The greatest range of relative density deviations is observed at high altitudes above the south pole (Fig. 2). Otherwise the seasonal range of density is generally less in the Southern than in the Northern Hemisphere (Figs. 2 and 12), partly as a result of smaller temperature changes there. At first glance it seems odd that seasonal density changes are so small below 16 km at the south pole, except in the 1–2 km layer next to the surface where the extreme cold of winter causes pressures to be higher than normal (Figs. 2 and 8). Apparently because of the increase in cyclonic activity, pressure falls enough in winter between 4 and 16 km altitude to counteract the density increase which might otherwise be expected to occur from the winter temperature fall (see Figs. 2 and 8). This minimal density range extends as far north as Ushuaia. The pertinent temperature, pressure, and density values for Amundsen-Scott are listed below:

	Mean June (winter)			Mean December (summer)		
Altitude (km)	Temperature (K)	Pressure (mb)	Density (kg m^{-3})	Temperature (K)	Pressure (mb)	Density (kg m^{-3})
5	228.2	489.92	0.74793	236.6	503.84	0.74187
8	209.8	306.40	0.50878	219.6	320.79	0.50891
11	203.7	185.94	0.31800	224.1	201.52	0.31327
15	196.7	94.11	0.16668	232.4	111.05	0.16646

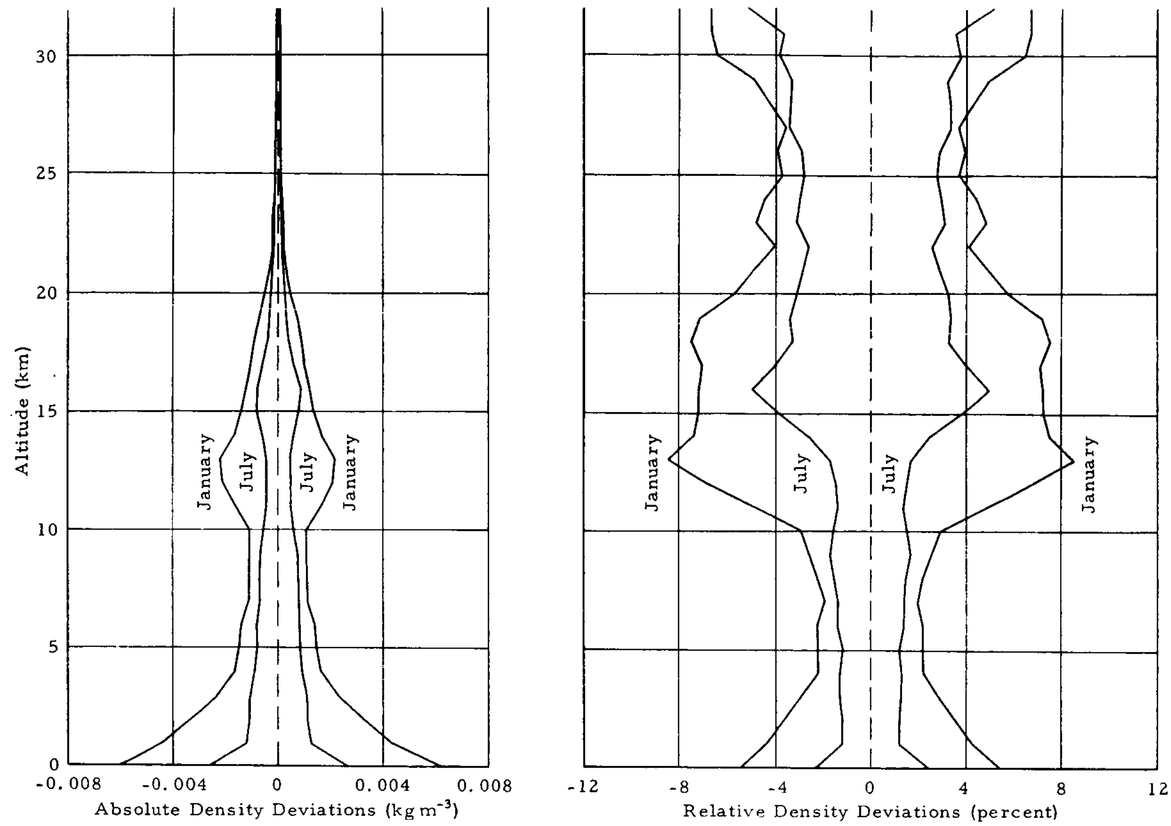

FIG. 10. ±3-Standard deviation of density envelopes for January and July from the median annual density at Cape Kennedy (Canaveral), Fla.

Deviations of density in the layer of maximum density deviation, as in the isopycnic layer, do not compare readily latitudewise, because of height changes within the layer. Temperature and pressure along these levels are difficult to compare for the same reason. However, it should be stated that the mean pressure of the isopycnic layer near the poles is about 400 mb, but drops to near 200 mb at the equator, contrary to statements by Sissenwine, Cole and Ripley (1958), and some earlier investigators that the "dividing level" (i.e., isopycnic level) generally occurs at 400- to 500-mb pressure.

5. The second isopycnic layer

The existence of a second isopycnic layer has been suggested by several investigators. Sissenwine, Ripley and Cole (1958) investigated density variations at several stations, from Tampa, Fla., to Thule, Greenland, and reported an isopycnic level at 8 km at all stations, with indications of a second level near 26 km. Quiroz (1961) suggested several high altitude isopycnic levels for middle latitudes. Cole (1961) suggested an isopycnic level, or a level of minimum density variation in the 80–90 km region over Churchill. Whitehead, Pitts and Blick (1963) suggest a number of isopycnic layers up to 90 km. The relative density deviation graphs (Fig. 2) indicate several short isopycnic layers. The more detailed study of density at Cape Kennedy from 7 years of record (Figs. 6 and 9) shows such a layer near 18 km. Neither the absolute nor the relative annual ±3-standard deviations of density graphs at Cape Kennedy (Fig. 10) show this layer, partly because it varies in altitude at different seasons; but the monthly density deviations (Fig. 9) show it plainly, and it can also be located in Fig. 6.

At Cape Kennedy the second isopycnic layer appears to be associated with temperature changes near the tropopause. It rises in winter when the tropopause rises and falls in summer when the tropopause falls, but it occurs where the maximum temperature deviations occur near the tropopause (shown by Smith, 1963) rather than at the tropopause itself.

There is also a very weak isopycnic layer at about 23 km above the equatorial zone. This can be seen in the Guayaquil cross section (Fig. 7). It is associated with the periodic temperature variations at that level reported by Smith (1963). Another isopycnic layer is found near 14 km altitude at the south pole (Figs. 2, 3 and 8), and from Fig. 5, it appears that a weak isopycnic layer may occur near 21 km at Buffalo. The height of this second isopycnic layer, where known, is shown by the dotted line on Fig. 3. It seems likely that additional isopycnic layers may also exist at higher levels wherever seasonal reversals of temperature and pressure occur. For further study of isopycnic layers see Figs. 4, 5, 6, 7 and 8.

An interesting study of the relations between temperature, pressure density changes can be made by comparing the Cape Kennedy density cross section (Fig. 6) and the Cape Kennedy pressure cross section (Fig. 11) with the Cape Kennedy temperature cross section shown by Smith (1963). It will be readily apparent that pressure is the major factor in determining density. This is also true from theoretical considerations, for under most conditions, temperature variations are only a small fraction of the total absolute temperature at any one point. In general, density is below average where pressure is below average, and above average where pressure is above average, but at Cape Kennedy it can be seen that the pressure deviations are modified by temperature deviations to help produce the isopycnic lines.

The pressure and density changes above the tropopause at Cape Kennedy show some tendency to propagate downward as the season advances, as does temperature (Smith, 1963), but the downward propagation of pressure and density is not nearly so marked as in the case of temperature. That the isopycnic lines should propagate downward in the equatorial stratosphere (see Fig. 7) is not unexpected. Reed (1962) and others have shown that tropical wind reversals at the higher altitudes likewise propagate downward. Whether this will occur in a 26-month cycle as shown for wind by Reed (1962) cannot be determined from the International Geophysical Year data, but such a study should be rewarding. At Guayaquil (Fig. 7) and Amundsen-Scott (Fig. 8) density seems to propagate downward at all levels.

Extreme values of density should be of interest, but such data are rarely published. Computations, using the lowest pressures reported in severe tropical hurricanes, indicate that under extreme conditions, density may fall to 1.0 kg m^{-3} at sea level. At the other extreme a sea level density of 2.0 kg m^{-3}, may sometimes occur in Siberia. Density decreases almost exponentially with height and at 30 km the mean density from the International Geophysical Year data along the 80th meridian west is less than 0.02 kg m^{-3}. This agrees closely with the U. S. Standard Atmosphere (1962) which gives a density of 0.0184 kg m^{-3} at 30 km. Due to measurement difficulties, extreme densities at high altitudes cannot be accurately determined, but relative variations are large (see Figs. 2 and 4–10), and it appears probable that values below 0.01 kg m^{-3} may occur at 31 km altitude.

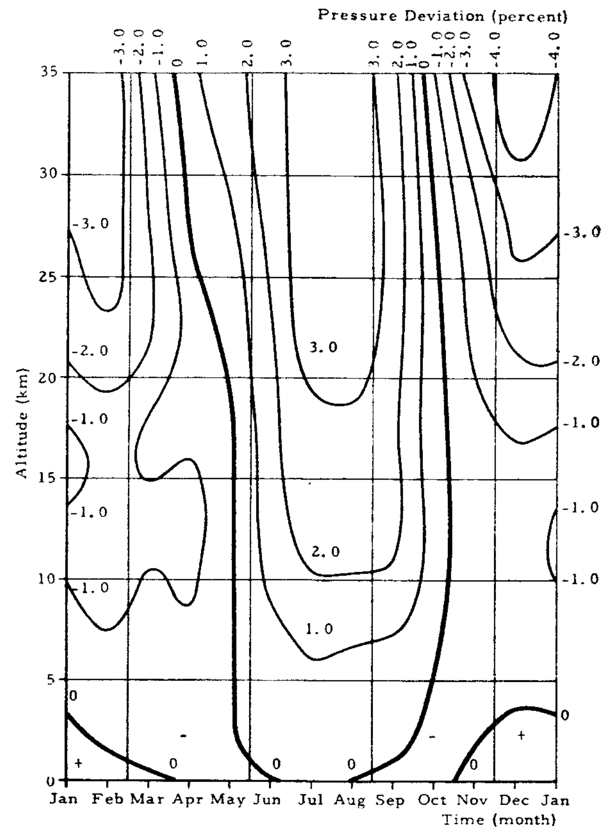

FIG. 11. Relative deviation of pressure from the median annual pressure at Cape Kennedy (Canaveral), Fla.

6. Conclusions

From the preceding analysis it is evident that variations of air density are so large with respect to season and latitude that no single standard atmosphere, presenting density as a function of altitude only, can present density data adequate for high speed vehicle operations on a global basis. Furthermore, it has been illustrated that the isopycnic layers and the maximum density deviation layer vary considerably in altitude as a function of latitude.

Acknowledgments. This study would not have been possible without the basic data which were computed under the direction of O. E. Smith. The writer is indebted to W. W. Vaughan for editing the rough draft and to other members of the Aerophysics Branch for helpful suggestions.

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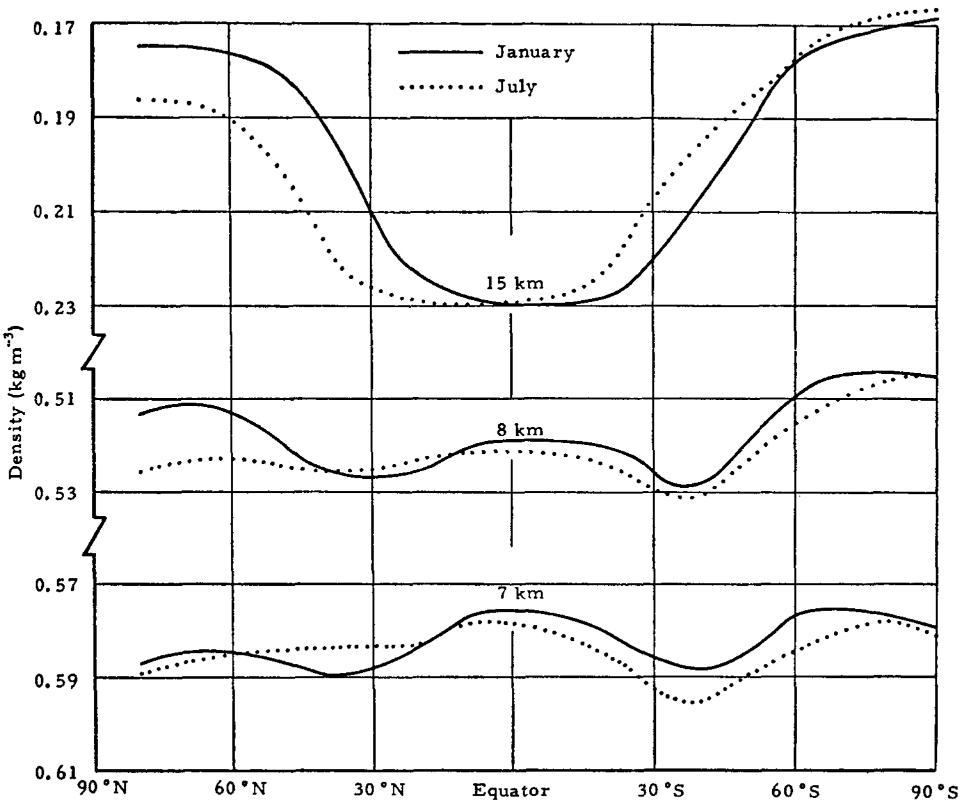

FIG. 12. Seasonal and latitudinal density variations at 7, 8 and 15 km.

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